

XIV. *A Letter from the Rev. Mr. Richard Dunthorne to the Reverend Mr. Richard Mafon F. R. S. and Keeper of the Woodwardian Museum at Cambridge, concerning the Acceleration of the Moon.*

S I R,

Cambridge, Feb. 28, 1748-9.

Read June 1.
1749.

AFTER I had compared a good Number of modern Observations made in different Situations of the Moon and of her Orbit in respect of the Sun, with the *Newtonian* Theory, as in my Letter of Nov. 4, 1746;† I proceeded to examine the mean Motion of the Moon, of her Apogee, and Nodes, to see whether they were well represented by the Tables for any considerable Number of Years, and whether I should be able to make out that Acceleration of the Moon's Motion which Dr. *Halley* suspected. *Vide* Phil. Transf. n. 218.

To this End I compared several Eclipses of the Moon observed by *Tycho Brahe*, as they are set down in his *Progymnasmatæ*. p. 114, with the Tables*, and found them agree full as well as could be expected; considering the Imperfection of his Clocks, and the Difficulty there must commonly have been in determining the Middle of the Eclipse from the Facts observed, as published in his *Historia Cælestis*. Indeed the small Distance of Time between *Tycho Brahe* and

† See these Transact. No. 482. p. 412.

* My Tables corrected as in my former Letter; which is always to be understood of the Tables mention'd in this.

and *Flamsteeda* render'd *Tycho's* Observations but of little Use in this Enquiry.

The next Observations that occurred to me were those of *Bernard Walther* and *Regiomontanus*, which being at double the Distance of Time from *Flamsteed* that *Tycho's* were, seemed to promise some Assistance in this Matter: Upon comparing such of their Eclipses of the Moon whose Circumstances are best related with the Tables, I found the computed Places of the Moon were mostly 5' too forward, and in some considerably more, which I could hardly persuade myself to throw upon the Errors of Observation; but concluded, that the Moon's mean Motion since that time, must have been something swifter than the Tables represent it; though the Disagreement of the Observations between themselves is too great to infer any thing from them with Certainty in so nice an Affair.

Then I compared the four well-known Eclipses observed by *Albategnius* with the Tables, and found the computed Places of the Moon in three of them considerably too forward: This, if I could have depended upon the Longitude of *Araña*, would very much have confirmed me in the Opinion, that the Moon's mean Motion must have been swifter in some of the last Centuries than the Tables make it; though the Differences between these Observations, and the Tables, are not uniform enough to be taken for a certain Proof thereof.

I could meet with no Observations of Eclipses to be at all depended upon between those of *Regiomontanus* and *Albategnius*, except two of the Sun and one of the Moon made at *Cairo* in *Egypt*,

related in the *Prolegomena* to *Tycho Brahe's Historia Cœlestis*, p. 34; nor any between those of *Albatagnius* and *Ptolemy*, besides the Eclipse of the Sun observed by *Theon* at *Alexandria*; notwithstanding I carefully searched all the Remains of Antiquity I could find with that View. These Eclipses of the Sun are the more valuable, because they were observed in Places the Longitudes and Latitudes whereof are determined by Monsieur *Chazelles* of the *Royal Academy of Sciences*, who was sent by the *French King* in the Year 1693, with proper Instruments for that Purpose. *Du Hamel Hist. Acad.* p. 309, 395.

The solar Eclipse observed by *Theon* was in the 112th Year of *Nabonassar* the Day of *Thoth*, according to the *Egyptians*, but the 22d Day of *Pauni*, according to the *Alexandrians*: He carefully observed the Beginning of 2 temporal Hours and 50' Afternoon, and the End at 4½ Hours nearly Afternoon at *Alexandria*. *Theonis Comment. in Ptol. mag. Construct.* p. 332. This Eclipse was June 16, in the Year of Christ 364: And the temporal Hour at *Alexandria* being at that time to the equinoctial Hour as 7 to 6, makes the Beginning at 3 equinoctial Hours and 18' Afternoon, and the End at 5 equinoctial Hours 15' nearly.

The Eclipses observed at *Grand Cairo* were as follows.

“ Anno Hegiræ 367, die *Jovis*, qui erat 28, rabie
 “ posterioris (is est ordine mensis quartus, et incipit
 “ ille annus *Saracenicus* die 19 *Augusti*, anno *Christiano* 977) observatum fuit *Cahiræ* in *Ægypti*
 “ metropoli initium eclipsis solaris, cum altitudo solis
 “ esset

“ effet $15^{\circ} 43'$. quantitas obfcurationis 8 digit. Ea
 “ finita, fol elevabatur $33\frac{1}{2}gr.$ *Ex Schickardo in*
 “ *MS.*” This Eclipse was *Decemb.* 13, in the
 Year of Chrift 977, the Beginning at $8^h 25'$, and the
 End at $10^h 45'$ apparent Time in the Morning.

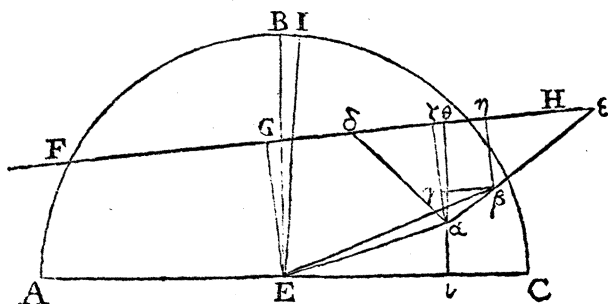
“ Anno eodem die Sabbathi, videlicet 29 mensis
 “ *Sywal* (numero decimi, qui Pafchalis eft eorum)
 “ eclipsis Solis occupavit digitos $7\frac{1}{2}$. In principio
 “ Sol altus fere 56° . In fine Sol occiduus elevaba-
 “ tur gradibus 26. *Ex Schickardo in MS.*” —
 This Eclipse was *June* 8, in the Year of Chrift 978.
 The Beginning at $2^h 31'$, and the End at $4^h 50'$
 apparent Time Afternoon.

“ Anno Hegiræ 368 (qui incoepit die 9 *Augufti*,
 “ anno *Chriftiano* 978) die *Jovis*, 14 *Sywal*, Luna
 “ fuit orta cum defectu, qui ad $5\frac{1}{2}$ digitos accrevit ;
 “ cum extaret fupra horizontem gradibus etiam 26
 “ fubaudio finem tunc accidiffe). *Schickardus.*” ----
 This Eclipse was *May* 14, in the Year of Chrift 979 ;
 but as the Middle cannot be known from what was
 obferved of it, I made no ufe thereof in this Enquiry.
 The Account concludes with the following Para-
 graph :

“ Hæ tres obfervationes habitæ funt ab *Ibn-Junis*,
 “ qui juffu Regis *Abu-Haly Almanzor*, fapientis,
 “ *Egypto* tunc Imperantis, rebus vacabat cœlef-
 “ tibus. Hujus authoris tabulas habet *Jac. Golius*
 “ Profefſor *Lugdun.* (qui mihi inde communicavit
 “ ifta) in quibus plures aliæ, fui et ſuperioris ævi ob-
 “ ſervationes extant. Locus obfervationis propinquus
 “ urbi *Cahiro.* *Schickardus.*”

That

That the before-mention'd solar Eclipses might be applied to the Examination of the Lunar Motions, I contriv'd the following Method ; which I think renders Eclipses of the Sun as useful at least as those of the Moon are in that Business.



Let ABC in the annexed Figure represent half the Earth's enlightened Disk, AEC a Portion of the Ecliptic projected thereon FGH the Path of the Moon's Shadow over the Disk, EI , the universal Meridian, α the Situation of the Place at the Beginning of the Eclipse, β its Situation at the End thereof, δ the Centre of the Shade at the Beginning, and ϵ its Centre at the End of the Eclipse. Draw EG , $\alpha\zeta$, and $\beta\eta$, perpendicular to the Path of the Shadow, $\beta\gamma$ parallel thereto ; join $\alpha\delta$ and $\beta\epsilon$, and through α draw $\theta\alpha\iota$ perpendicular to AC .

Then (computing the true Places of the Sun and Moon at the observed Times of the Beginning and End of the Eclipse) we shall have given $\delta\epsilon$ the Motion of the Moon from the Sun in her Orbit during the Time of the Eclipse, and $\alpha\delta = \beta\epsilon$ the Semidiameter of the *Penumbra* ; which are to be reduced into such Parts as the Semidiameter of the Disk contains

tains 10000: The Angles BEI and BEG , being found by Methods commonly known, GEI their Sum or Difference will be likewise given. Also $E\alpha$ and $E\beta$ will be Sines of the Sun's Altitude at the Beginning and End of the Eclipse respectively; $IE\alpha$ and $IE\beta$ are the Angles at the Sun between the Vertex of the Place and the Pole of those Times; which being found, the Angle $\alpha E\beta$, their Difference will be known, from whence the Line $\alpha\beta$ and the Angle $E\alpha\beta$ may be computed.

The Angle $GE\alpha$ is the Sum or Difference of the known Angles GEI and $IE\alpha$: In the Figure before us, the Complement of this to a Semicircle is $E\alpha\gamma$; which being subtracted from $E\alpha\beta$ leaves the Angle $\gamma\alpha\beta$, from whence and the Line $\alpha\beta$, $\alpha\gamma$, and $\gamma\beta = \zeta\eta$ may be found.

Let $a = \delta\epsilon - \zeta\eta$, $b = \alpha\delta = \beta\epsilon$, $c = \alpha\gamma$, and $x = \beta\eta = \gamma\zeta$.

Then $\sqrt{bb - xx} = n\epsilon$, and $\sqrt{bb - cc - 2cx - xx} = \delta\zeta$, by *Eucl.* 1.47.

Consequently $a - \sqrt{bb - xx} = \sqrt{bb - cc - 2cx - xx}$ which being reduced, gives us the quadratic Equation $xx - cx = \frac{4a^2b^2 - a^4 - 2a^2c^2}{4aa + 4cc}$. This Equation solved,

gives us the Value of x , from which $\delta\zeta$ and $n\epsilon$ will be likewise had. In the Triangle $\alpha\zeta\theta$ we have $\alpha\zeta$ and the Angle $\zeta\alpha\theta = GEB$ given, whence $\alpha\theta$ and $\zeta\theta$ may be found: Consequently $\delta\theta$ will be known; and from the observed Time of the Beginning of the Eclipse, and hourly Motion of the Moon from the Sun, the Time when the Centre of the Shade is at θ will be had. Lastly, in the Triangle $E\alpha\iota$, we have given the Side $E\alpha$, and the Angle $E\alpha\iota = BE\alpha$

BE α (the Sum or Difference of the Angles BEI and IE α); therefore the Sides EI and αI may be found. But EI is the Distance of the Moon from the Sun in the Ecliptic, and $\alpha I \mp \alpha \theta$ the Moon's Latitude at the Time when the Centre of the Shade is at θ ; which may be compared with the Computarion from the Tables for that Time.

By this Means I compared the aforesaid Solar Eclipses with the Tables, and found the Difference in Longitude and Latitude, as follows.

A.D.	Apparent Time at Greenwich.		Dist. γ a \ominus from EI.	Lat. γ from θ .	γ a \ominus by Tab.	Lat. γ by Tab.	Diff. from Obser.		Diff. in Lat. from Digits observed.
							nLong	in Lat.	
364	June 16.	3 4 20	39 41 in conseq.	34 37 Nor.	35 25 37	26 Nor.	- 4 16	+ 2 49	
977	Dec. 12.	19 12 30	43 39 in antec.	30 23 Nor.	36 3 31	50 Nor.	+ 7 36	+ 1 27	- 2 36
978	June 8.	1 16 12	29 3 in conseq.	8 24 Sou.	37 48 3	21 Sou.	+ 8 45	- 5 3	+ 3 38

The Agreement there is between the two last of these Differences in Longitude, shews that the Tables represent the mean Motion of the Moon's Apogee very well for above 700 Years, the Moon being very near her Perigee at the Time of one of those Eclipses, and near her Apogee at the Time of the other.

By the same Method I also compared the Sun's Eclipse, *July 29, 1478.* (which appears, from what is related of it, to have been carefully observed by *Bernard Walther* at *Nuremberg*), with the Tables, and found the Difference in Longitude to be $+ 10' 29''$, and in Latitude $+ 9' 12''$. This wide Difference in Latitude, from the Tables, that agree so well with the former ancient Observations, confirmed me in the Opinion, that the *Nuremberg* Obser-

Observations are too inaccurate to determine any thing from them in this Affair.

The Eclipses recorded by *Ptolemy* in his *Almagest*, are most of them so loosely described, that, if they shew us the Moon's mean Motion has been accelerated in the long Interval of Time since they happened, they are wholly incapable of shewing us, how much that Acceleration has been. There are indeed two or three of them attended with such lucky Circumstances as not only plainly prove, that there has been such an Acceleration, but also help us to guess at its Quantity. One of these is the Eclipse, said by *Hipparchus* to have been observed at *Babylon*, in the 366th Year of *Nabonassar*, the Night between the 26th and 27th Days of *Thoth*, when a small Part of the Moon's Disk was eclipsed from the North East, half an Hour before the End of the Night, and the Moon set eclipsed. This was in the Year before Christ 313, *Decemb. 22*. The Middle of this Eclipse at *Babylon* (supposing with *Ptolemy* the Meridian of that Place to be 50' in Time East of the Meridian of *Alexandria*), by my Tables was *Dec. 22. 4^h 4' apparent Time; the Duration was 1^h 37', *Ptolemy* makes it 1^h 30' nearly; whence the Beginning should have been about 5^h 15' after Midnight: According to *Ptolemy*, the Night at *Babylon* was at that Time 14^h 24' long, and therefore Sun rise at 7^h 12' after Midnight; and as the Moon had then South Latitude, and was not quite come to the Sun's Opposition, her apparent Setting must have been something sooner, *i. e.* more than an Hour before the Beginning of the Eclipse, according to the Tables; whereas the Moon was seen*

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eclipsed

eclipsed some Time before her Setting; which, I think, demonstrates, that the Moon's Place must have been forwarder, and consequently her Motion since that Time less than the Tables make it by about 40' or 50'. But the computed Place of the Moon in each of the before-mentioned Solar Eclipses observed at *Grand Cairo*, being about 8' before her Place, from Observation shews us, that the mean Motion of this Luminary has been something greater in the last 700 Years than the Tables suppose it, and therefore must have been accelerated.

This Acceleration is further confirmed by the Eclipse, which *Hipparchus* says was observed at *Alexandria*, in the 54th Year of the second *Calippic* Period, the 16th Day of *Messori*, when (he says) the Moon began to be eclipsed half an Hour before her Rising, and was wholly clear again in the Middle of the third Hour of the Night. This was in the Year before Christ 201. *Sept. 22.* The Middle of this Eclipse at *Alexandria* by the Tables was *Sept. 22. 7^h 44'* apparent Time; and the Duration 3^h 4', which makes the Beginning at 6^h 12' apparent Time, that is, about 10' after the rising of the Moon at *Alexandria*, or 40' later than the Beginning from Observation. This Difference in Time makes a Difference of near 20' in the Moon's Place.

The most antient Eclipse of which we have any Account remaining, namely that related by *Ptolemy*, to have been observed at *Babylon* the first Year of *Mardokempad*, in the Night between the 29th and 30th Days of *Thoth*, in which the Moon began to be eclipsed when one Hour after her Rising was fully past; if, by reason of the Latitude of the Expression,

pression, it be not a direct Proof of the Acceleration, it may nevertheless help to limit its Quantity. This Eclipse was in the Year before Christ 721. *March 19.* The Middle whereof at *Babylon*, by the Tables, was *March 19.* $10^h 26'$ apparent Time; and the Beginning at $8^h 32'$, the apparent Rising of the Moon at that Place was about $5^h 46'$ Afternoon; so that the observed Beginning of the Eclipse was at least $6^h 46'$ Afternoon, *i. e.* not above $1\frac{1}{4}^h$ before the Beginning, by the Tables: Wherefore the Moon's true Place could precede her Place by Computation but little more than $50'$ at that Time.

If we take this Acceleration to be uniform, as the Observations whereupon it is grounded are not sufficient to prove the Contrary, the Aggregate of it will be as the Square of the Time: And if we suppose it to be $10''$ in 100 Years, and that the Tables truly represent the Moon's Place about *A. D.* 700. it will best agree with the before-mentioned Observations; and the Difference between the Moon's Place by the Tables and her Place in the Heavens, will be as follows.

Years before Christ.	Error of Tab.	Years of Christ.	Error of Tab.	Years of Christ.	Error of Tab.
7 0	—56 6	200	—12 30	1100	+ 4 0
600	—49 50	300	— 9 20	1200	+ 4 10
500	—44 0	400	— 6 30	1300	+ 4 0
400	—38 30	500	— 4 0	1400	+ 3 30
300	—33 20	600	— 1 50	1500	+ 2 40
200	—28 30	700	0 0	1600	+ 1 30
100	—24 0	800	+ 1 30	1700	0 0
A.D.O	—19 50	900	+ 2 40		
100	—16 0	1000	+ 3 30		

I am,

S I R,

Your humble Servant,

Richard Dunthorne.

XV. Alberti Halleri, *Archiatri Reg. Medicin. Prof.* Gotting. & R. S. Lond. *S. Fabricæ morbosæ in cadaveribus repertæ historiæ aliquæ.*

O B S. I.

Read, June 8. 1749. **I**N femina quadragenaria reperi Venam cavam inter renalis sinistræ originem, et inter iliacas venas, enormiter angustatam, ut vix quid-